# The relationship between working time and productivity - Intensity of labour 

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Abstract: Influential economists like Marx, Jevons, Chapman and Robbins argued that working time and labour intensity - productivity are strongly correlated. Nevertheless, the quantitative relationship between these two magnitudes has never been estimated. This article makes a step towards estimating this relationship. It proposes a new theoretical formulation, following the ideas of influential economists. It utilizes for the first time data from ergonometric experiments to estimate the material basis of this relationship. The estimated relationship between working time and maximum intensity for manual labour is mainly based upon the physical limitations of human body. The findings confirm the thesis of a strong negative correlation that can lead to output maximization following a working time reduction. They can also be used to explain the long term decreasing trend of working time that seems to reach an end in our days.

Keywords: Working time, labour intensity, labour productivity.

## A Relação entre tempo de trabalho e produtividade - Intensidade do trabalho

Resumo: Economistas influentes como Marx, Jevons, Chapman e Robbins argumentaram que o tempo de trabalho e a intensidade do trabalho estão fortemente correlacionados. No entanto, a relação quantitativa entre essas duas magnitudes nunca foi estimada. Este artigo dá um passo no sentido de estimar esse relacionamento. Ele propõe uma nova formulação teórica e utiliza, pela primeira vez, dados de experimentos ergométricos para estimar a base material desse relacionamento. A relação estimada entre tempo de trabalho e intensidade máxima do trabalho manual baseia-se principalmente nas limitações físicas do corpo humano. Os resultados confirmam a tese de uma forte correlação negativa, que pode levar à maximização da produção após uma redução do tempo de trabalho. Eles também podem ser usados para explicar a tendência decrescente de longo prazo do tempo de trabalho.

Palavras-chave: Tempo de trabalho, intensidade do trabalho, produtividade do trabalho.

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## 1. Introduction

The tendency of working time to fall is one of the longest recorded trends in economic history, invigorating the widespread perception that it is irreversible. However, there is evidence that working time has stopped decreasing in the United States since the mid-1980s while in Europe the declining pace seems to have reached an end too ${ }^{1}$.

The explanation of working time movements on the basis of individual choice or class struggle alone is in our opinion incapable of explaining the recent developments on this issue. There is a need for a broader explanation. We believe that working time never stopped being an important factor of capital accumulation. Its decreasing trend has led to its theoretical depreciation. This wouldn't have happened if working time was not detached from the other dimension of labour-power expenditure: the intensity of labour. We maintain that the correlation of these two dimensions can shed light to the role of working time in economy and to its historical evolution.

In the past, Marx and also influential neoclassical economists like Jevons, Chapman and Robbins, emphasized the importance of the intensive dimension of labour. According to them, the existing of an inverse relationship between working time and labour intensity (productivity in a different terminology) was so decisive that they even expected an output increase after a working time decrease. They have attributed a major role to this correlation, despite the fact that it was impossible for them to proceed further, since the time - intensity relationship was impossible to be estimated.

The quantitative determination of this relationship is still unaccomplished. The difference between mental and manual labour, as well as the differences among various kinds of work processes make this task even harder. But since the determination of this relationship is crucial for understanding major aspects of capitalism's modus operandi, a quantitative modelling is necessary even under limiting assumptions. A step towards this direction is the main objective of this article. In order to address the problem, we use data from ergometric experiments simulating manual labour, since other kind of data are either problematic or absent. We study the secular base of the contradiction between working time and labour intensity, which is humans' limited capability for energy expenditure. This does not mean that we underestimate workers' discretion or other social factors affecting labour intensity determination. On the contrary, we maintain that the estimated relationship among working time and (manual) labour intensity provides a material base which describes the boundaries within which workers and capitalists exert their discretion or authority.

In our technical model, we follow to some extent but also significantly modify the approach developed by Barzel (1973) to this problem. Our target is, first to construct a theoretical model for the time - intensity relationship and then to estimate this relationship.

In the end it can be argued, among others, that the historical decline of working time provided the capability and most probably led (ceteris paribus) to an increase in total output. So, working time decrease accompanied by intensity increase, served not only workers but capital's interests as well.

The paper is structured as follows. First, we briefly present the basic ideas of influential economists on the time - intensity correlation and the workers' effort discretion. Then we describe a model that can incorporate the main features of this relationship, in comparison and contradiction to previously presented models. Third, with the help of

[^1]modern ergonometric experiments we estimate the relationship between working time and intensity for manual work. Finally, we explore some initial economic implications and limitations of this relationship and conclude.

## 2. The evolution of economic thought on the time - intensity relationship

The belief that the relationship between working time and labour intensity affects the production conditions is quite old in Economic science. In this section we present some basic ideas on this relationship that will subsequently help us construct our model.

Marx, in his basic surplus value analysis assumed the intensity of labour to be a stable quantity, irrespective of the working time fluctuations, but it was clear that this was a simplifying assumption at the highest level of abstraction. In different parts of 'Capital' Marx argues that there is a strong correlation among these two variables. The 'self - evident law that the efficiency of labour power is in inverse ratio to the duration of its expenditure' (Marx 1990, p.535) is founded, according to Marx, on the secular limitations that nature poses to human bodies and minds. This contradiction was making the consumption of the labour power for both long hours and high intensity, not possible. Marx also argues that this inverse relationship was not only theoretical but it was present in the real work process. He refers to many examples proving that the reduction of working time in real life has led to major increases in labour intensity (Marx 1990, p.533-542). In his opinion, as the factory discipline and the extended use of machines had already made workers to reach their physical limits, it was impossible to achieve any further intensity increase without shortening the working day.

Another important aspect of Marx's opinion on this matter is that he didn't focus on the daily variation of labour intensity. He argues that since work is not a daily paroxysm but a repeated day after day process, it is this recurrent nature that is creating the trade-off between working time and labour intensity. This means that the daily fluctuation of the intensity of labour is a secondary phenomenon, while the primary is the transfer of fatigue from one working day to the following(s).

As will be shown later, the above formulation has a decisive advantage: it incorporates the possibility to observe a total output decrease after a working time increase, without assuming negative productivity of labour.

Although a detailed analysis of Marx's treaty on this topic is not the intention of this article (see also loannides \& Mavroudeas, 2011), it is our belief that Marx based the above described inverse relationship both on 'objective' or secular factors like the human organism limits and on 'consciousness' or 'subjective' factors as well, such as the determination through class struggle. His methodological point of view was materialistic, trying to explain social phenomena after describing the materialistic ground upon which they were developing. Following this approach, we argue that the biological limitations of human organism (the ability of human body to increase working time only at the price of reducing intensity) provide the material basis upon which the process of working time determination, through class struggle, is taking place.

It is quite interesting that Marx's view on this issue was shared by influential neoclassical economists that lived many years after him. Chapman (1909) relates the intensification of labour to the use of machines. He points out that output increased after working time reductions, because of the subsequent increase in intensity.

Following a parallel reasoning, Robbins (1929) argues that the reason for the fall in workers' productivity that follows an increase in working time is the repeated nature of
labour and the accumulation of fatigue. There is an obvious similarity with the corresponding Marxian argument. He also extents this result to the workers of mind. Most importantly, he maintains that there is an optimum (not the maximum) length of the working day that leads to output maximization.

Even before them, Jevons himself set the task of quantifying the time - intensity relationship. He compared the effectiveness of the marginal reward given to a worker compared to the marginal effort he can offer, given that any additive effort comes on a higher price of workers fatigue and exhaustion. For this reason, he conducted experiments with weights, trying to determine how long a person can uphold a certain weight. These experiments were an interesting attempt to connect working time with intensity and work fatigue. As he mentioned: "In a regular and constant employment the greatest result will always be gained by such a rate as allows a workman each day, or each week at the most, to recover all fatigue and recommence with an undiminished store of energy" (Jevons, 1888, p.209).

The time - productivity relationship has been investigated by more recent neoclassical authors too. Feldstein (1967), Barzel (1973) and Ehrenberg (1971) and more recently, Contensou and Vranceanu (2000) have questioned the standard 'homogeneity assumption' (i.e. that labour has the same impact on output independently of its internal composition of number of workers and hours of work). They proposed various mathematical models to describe the time - intensity relationship. Their main conclusion is that working time has an impact on labour productivity that stems mainly from the physical limits of the human body and mind and not from workers' will. Nevertheless, they didn't proceed to a quantitative description of the relationship between working time and intensity.

In recent years, there is a growing and already quite extended literature examining the effects of working time on productivity - intensity (these terms are used equivalently, see below). There is a variety of surveys correlating long hours to workers physical and mental health and also with work injuries and increased mistakes during labour time that all tend to diminish labour productivity in the long run (Sparks et al 1997; Dembe et al., 2005; Burke, 2009; Ricci et al., 2007). According to this line of research, long hours have an indirect effect on productivity, mainly through workers health, injuries or mistakes. They constitute an additional supportive basis to our investigation of the direct effect of working time to the productivity - intensity of labour. Finally, a big part of the literature is investigating the results of flexible working time arrangements on productivity (for a comprehensive presentation see Golden 2011). Although not directly related to our investigation, it should be noticed that an outcome of this literature is that the negative effect on productivity resulting from the deviation of workers' preferred hours compared to actual hours is growing when working time is longer.

In a different direction from the above, there are studies, many of them recent, which directly measure the physiological basis of the time - intensity relationship. Among them, Cette et al. (2011) find, with the help of two extended panel data sets for 18 OECD countries over a long time period (1870-2005), that hourly productivity is a diminishing function of working time and that the negative elasticity of productivity towards working time is bigger (in absolute values) the bigger the working time. Similar arguments were made by other authors with Leslie and Wise (1980), Shepard and Clifton (2000), Holman et al. (2008) among them. The majority of this strand of the literature concludes that productivity is decreasing on working hours. Cette et al (2011) specifically examine three
time thresholds, those of 1825, 1925 and 2025 annual working hours. Their conclusions are very interesting and confirm our results that will follow, although through a totally different method. They estimated that:
... .A $1 \%$ increase in working time would lead to a decrease in productivity of roughly $0.9 \%$ for the threshold of 1925 hours and of $1 \%$ for the threshold of 2025 hours. This also suggests that, given the very high initial levels of hours worked, the reduction in output stemming from decreasing working time would be mostly offset by the productivity gains associated with the decrease in working time (Cette et al., 2011).

The above means that when working time is close to 40 hours per week (which is the equivalent of 2025 hours per year, assuming five weeks annual leave), an increase in working time is expected to have no effect on output.

A portion of the data utilized by this line of research derives from experiments that were conducted in some factories many years ago. Unfortunately, these experiments ceased to be conducted in our days. Contemporary data are analyzed as well but only on limited types of workers and jobs. Some of their findings are especially interesting. Pencavel (2015) for example has utilized available data for munitions workers during world war one in England. He estimated that above a time threshold, any increase in working hours has no effect in output. He also confirmed that the absence of a day of rest during a week has negative effects not only in productivity but in total output as well.

An equally interesting research comes from Dolton et al. (2016). They use data from a famous experiment conducted in Hawthorne Western Electric plant in Chicago during the 1920s and 30s, when workers were asked to work at various time schedules, so to measure their product in each case. Dolton et al. (2016) derive a specific and strong conclusion, very similar to those of Pencavel (2015) and Cette et al. (2011), that any increase in working time beyond a certain threshold will not result in any increase in total product, but to the opposite. They also estimate this threshold, to find it approximately eight and a half working hours a day, a result very similar to ours (which will be presented below) although using a totally different methodology.

An example of an investigation that uses contemporary data is that of Collewet and Sauermann (2017), who examine workers in call centers that work part time. Although working less than eight hours a day, these workers are found to suffer a reduction in productivity when their (short) working time is increased. As the authors point out, this productivity decrease is expected to become stronger when working time is close to a full time working day. Another interesting point is that the management seemed to be aware of the fact that part timers are more productive than full timers, showing a clear tendency to employ the first category for the job.

Theories that focus on workers' will or consciousness as the main factor determining the intensity of labour follow a different path. Many authors, who recognized the importance of workers' effort in the labour process, attributed it on motives or pressures and other similar procedures. The effort discretion that workers enjoy (Leibenstein, 1979) and the roles of motives (mainly wage) in the determination of effort (Akerlof, 1976 and 1982; Schmidt-Sorensen, 1991) are two approaches, which ascribe intensity's determination on factors other than the limits of human organism. On the other hand, Green (2001) refers to a social process where intensity is determined either with a collective decision among workers or through class struggle. This social process ends in a commonly decided 'fair' intensity of labour.

Although these two paths may seem contradictory, in our opinion they can be complementary. Workers have the ability to influence the intensity of their labour; however, the power of both workers and employers is constrained by objective, material forces; In the workers' case, their collective or individual choice of labour intensity is mainly based upon the natural limits of human body and mind, given the technology of production. These natural limits put working time in contradiction to intensity and make workers to tend to work with less intensity when working time is long. On the other hand, given the material basis, managerial strategies (through incentives or fear of punishment) aim to force workers to come closer to these limits. In a nutshell, the material basis offers the ground and the constraints upon which the 'subjective' strategies of the social classes confront each other. Consequently, we maintain that the first necessary step for studying the factors determining labour intensity is to describe the inverse relationship between working time and intensity that is created by the limited capabilities of human organism. This is the aim of the rest of this article.

## 3. Modelling the time - intensity relationship

We will now proceed to the modelling of the working time - intensity relationship, following our previous work on this topic (loannides \& Mavroudeas, 2011). It is a common ground that working time affects labour intensity via two different ways. The first is the immediate one, inside a single working day. As working time passes, the accumulated fatigue reduces workers productivity - intensity for each extra working hour. So every working hour (except for a small time period at the beginning of the day) is worked less intensively from the previous one. The $3^{\text {rd }}$ working hour is less intensive from the $2^{\text {nd }}$, the $4^{\text {th }}$ from the $3^{\text {rd }}$ and so on. This is the so called 'exertion effect' that is well described among others by Barzel (1973). According to this approach, an increase in working time will lead to an intensity decrease but only for the increased working time. If for example, working time increases from 8 to 9 hours a day, the intensity of labour will decrease only for the ninth hour. The intensity of labour from the first hour to the eighth will remain unchanged. This is clear in the following figure provided by Barzel (1973) (Points A, B, and C, added by us).


Figure 1. Daily and hourly output
Source: Barzel (1973)

Barzel assumed that labour productivity is only affected by workers' effort. So he kept all the other factors that influence productivity constant. Under this assumption and with the appropriate adjustment of measurement units, the terms productivity and intensity of labour can be used equivalently, and in this way we use them throughout this article.

Secondly, according to figure 1, the exertion effect is active after point $A$, where productivity starts decreasing. Until this point, the inverse phenomenon ('warming up effect') is in action, which leads to a productivity increase. In our opinion, this phenomenon is overvalued in the above diagram, because it is only valid for a limited time fraction at the beginning of the shift when, due to adjustment to the work environment, productivity increases. Zero productivity at the beginning of the shift is also considered an extreme assumption with no practical consequences. Therefore, in the following figure 2, productivity starts from a non-zero value. We make both these assumptions aiming at a more realistic representation of the work process. The results of our analysis would not have changed if those assumptions were not made.

Ehrenberg's (1971) approach is similar. The contribution of each worker on output is measured by a productivity function $g(h)$, where $h$ is working time. He explicitly states that instant productivity cannot become negative, which means that as working time increases output will always increase. The major disadvantage of the above described approach is that it cannot examine and incorporate the effects of a working time increase for the whole working day and not only for the subsequent (to the increase) working hours. In our opinion, an increase in working time from B to C (figure 1) will not only reduce the intensity of labour for the time span BC but for the whole of the working day, OC. Of course this cannot happen inside the narrow limits of a single working day. The work that has already been done cannot be undone, so the intensity reduction can be realized inside a broader time period, since work is not a 'daily paroxysm' but has a repetitive nature. This mechanism of fatigue transfer from one day to the following(s) has been studied enough (see for example Dawson \& Fletcher, 2000) and is broadly accepted.

Since labour has a repetitive nature, fatigue is accumulated and transferred not only to the following working hours but to the following working days as well. So, the extended working time has a double effect. The first effect is that the intensity of labour will decrease for each extra working hour during that day. The second effect is that the whole time intensity relationship will change and this will lead to a reduction of the overall intensity for every hour of the following working days. The first effect is the one described by Barzel, but the second effect is not described in a formal way yet. As we can see in figure 2, Barzel's time - intensity relationship is described by the curve ABD (avoiding the rather extreme and meaningless assumption of zero productivity at the beginning of the day). Since $A B D$ is the intensity (which under our assumptions is equal to productivity) curve, output for working time $T_{1}$ equals the integral of the intensity function and is represented by the $0 \mathrm{ABT}_{1} 0$ area.


Figure 2. Feedback curve (instant)
Source: loannides and Mavroudeas (2011)

If working time increases from $T_{1}$ to $T_{2}$, then the productivity from time $T_{1}$ until time $T_{2}$ will fall according to Barzel and this decline is represented by the part BD. The second effect, which in our opinion is the crucial one, implies that the time - intensity relationship will change as a whole and not only for the part from $T_{1}$ to $T_{2}$. This means that during the following day(s), the initial productivity (intensity), at time $=0$ will not be A anymore but will be reduced to C. Intensity will decline (not necessary homogenously), for every point in time, shaping a new time - intensity curve CE, which attaches smaller intensity to every point in (working) time. A feedback phenomenon is occurring, from which this relationship is named the feedback relationship. So, when the working day is extended from $T_{1}$ to $T_{2}$ total output will now equals the $0 \mathrm{CET}_{2} 0$ area. There is no guarantee that this area $\left(0 C^{2} T_{2} 0\right)$ is bigger than the $0 A B T_{1} 0$, which was the output for working time $T_{1}$. This means that an increase in working time can probably lead to output reduction. And this will happen although instant productivity - intensity will always be positive.

For Barzel, any working time increase will lead to an output increase, unless the marginal product of labour becomes negative, i.e. a destruction of the product occurs. This is a rather extreme assumption that will rarely happen in reality and this is why no author makes it explicitly. Under the second approach (feedback relationship), it is possible to obtain an output reduction following an increase in working time (and the inverse as well).

Instead of using the instant intensity of labour as in figures 1 and 2, we can alternatively use the average intensity. We define as average intensity the intensity that, remaining stable during a working day, creates the same output as the fluctuating intensity (of both figures 1 and 2 ) for the same working time.

$$
\begin{equation*}
\varepsilon(t) \cdot T=\int_{0}^{T} i(t) d t=Q(t) \tag{1}
\end{equation*}
$$

Where $\varepsilon(t)$ is the average intensity, $i(t)$ is the (instant) intensity, $T$ is the duration of the working day and $Q(t)$ is the output as a function of time.

Equation (1) defines average intensity and also helps to calculate output when we use either instant or average intensity (on this we will return later). This is shown in Figure 3.


Figure 3. Average Feedback Curve
Source: Own estimations
$A B$ curve is the average intensity. As expected, the relationship between working time and intensity remains negative. Every increase in working time will lead to a decrease in average intensity (we neglect the warming up effect at this point). If working time is $\mathrm{T}_{1}$ the output now equals the area $0 \mathrm{EFT}_{1} 0$, since AB is the average intensity. If working time increases to $\mathrm{T}_{2}$ the new output equals the area $0 \mathrm{GHT}_{2} \mathrm{O}$. The average intensity curve can well represent both Barzel's and the feedback relationship. But in the first case (assuming Barzel's relationship) the $0 \mathrm{GHT}_{2} 0$ area will always be larger from the $0 \mathrm{EFT}_{1} 0$, when in the second case (feedback relationship) this is not assured. So it is the specific slope, or elasticity, of the curve that makes the difference between the Barzelian and the feedback relationship. It should be stressed here that the notion of average intensity can fully describe (without any loss of accuracy) the fact that the intensity of labour is not stable during a working day.

We use the notion of average intensity firstly for its simplicity. Apart from this, it is well known that many work processes, especially those related to machines, try to keep up a stable rhythm for various reasons. This is usually accomplished by setting machines to work in a standard speed. It has also been found that workers' fatigue is minimized when the speed of their movements is kept constant. So, average intensity is not only a mathematical sophistication but stems from the real work process as well.

## 4. The maximum acceptable work duration model

The estimation of the time - intensity relationship remains a very hard and still unaccomplished task. At this section we will try to make a step forward, with the help of an alternative, interdisciplinary method. As Bowles et al. (1983) note, we should put an ergometric device to each worker in order to achieve this task. One extra difficulty is the fact that the construction of a stable and comparable for all different kinds of jobs, measure of labour intensity is an ambiguous process. An indirect calculation of the intensity of
labour with the use of output is also statistically very complex, since many other factors such as the technology used, the methods of work organization, the different job tasks even in similar jobs, may influence total output. Most of these factors are hardly quantifiable or even classifiable and this makes extremely difficult to separate their effect on output. This is the main reason for the rarity of such studies.

Difficulties given, we turn to an alternative method, which is more abstract from an economic point of view, but more precise from a technical angle. Luckily enough, there is a scientific branch and a rather extended literature examining the working potential of human organism. This is done under laboratory conditions and the results can be used for our purpose. We refer to ergometric experiments aiming to examine the impact of working time on manual labour intensity. After all, Jevons himself (as mentioned earlier) conducted such experiments, trying to quantify the relationship between working time and intensity. In our days there is a modern extension of a rather old but pioneering ergometric research which is categorized as the 'maximum acceptable work duration model' (Drain et al., 2016). This line of research was initiated by the work of Birk et al. (1961) and Bonjer (1968). More recent studies such as Saha et al. (1979); Rodgers et al. (1986); Wu and Wang (2001 and 2002) kept on the same path. The most recent developments, according to Drain et al. (2016), on the general model of this methodology are offered by Wu and Wang (2002).

Under this methodology, the above scientists adapted ergometric devises to men and women and asked them to perform physical duties on an ergometric bicycle inside the laboratory. The setup of the experiments was such as to imitate the conditions of real manual working process. The similarity of these ergometric experiments with the real work process is well documented, so these experiments are suitable for the examination of manual work procedures. As Astrand (1956) and Lundgren (1946) among others report, a certain oxygen uptake gives the same heart rate on a bicycle ergo-meter as in manual labour which engages great muscular groups. That is why such a job can be fairly well reproduced, with regard to the load on the circulation and respiration, by work on a bicycle ergo-meter.

This bibliography has used three indices to describe the 'workload intensity' or 'physical workload' of every 'worker'. These are the 'maximum aerobic capacity' $\left(\% \mathrm{VO}_{2 \max }\right)$, the 'relative heart rate' $(\mathrm{RHR})$ and the 'relative oxygen uptake' $\left(\mathrm{RVO}_{2}\right)(\mathrm{Wu}$ \& Wang, 2002). For example, the 'relative heart rate' (RHR) is defined as follows:

$$
\begin{equation*}
\mathrm{RHR}=\left(\mathrm{HR}_{\text {work }}-\mathrm{HR}_{\text {rest }}\right) /\left(\mathrm{HR}_{\text {max }}-\mathrm{HR}_{\text {rest }}\right) \times 100 \% \tag{2}
\end{equation*}
$$

Where $\mathrm{HR}_{\text {work }}$ is the heart rate during work process, $\mathrm{HR}_{\text {rest }}$ is the heart rate in stable condition (not working) and $\mathrm{HR}_{\max }$ is the maximum heart rate that was measured for each 'worker'. The first interesting outcome of these experiments is that the intensity of labour is unsustainable for a significant time interval if it exceeds the $70 \%$ of the maximum intensity a human can provide. This is a proof that there are limits in the intensification process; limits imposed by the characteristics of human organism.

These experiments have also discovered that the 'relative oxygen uptake' and the 'relative heart rate' remain stable during manual labour process, until a specific point in time is reached. This means that human organism can adjust itself to a more or less stable working rhythm. After reaching this point (which can be extended from minutes till hours, due to labour intensity):
... an accumulation of lactic acid in the blood puts an additional load on the cardiovascular system and causes a sudden increase in heart rate. Thus, a markedly higher heart rate (about 10 beats per minute above steady state) towards the end of a work shift, as compared with the steady state heart rate observed during the initial hours of work, is a clear sign of fatigue" (Wu \& Wang, 2002, p.281).

Labour can be extended beyond this point but in this case fatigue is accumulating in workers' organism. The specific point in time where this change (exhaustion) in human organism takes place depends on the intensity of the working process. The bigger the intensity the sooner this point will come. This methodology allows for the calculation of a maximum working time which corresponds to every possible labour intensity. When surpassing this time, labour can be extended but at the price of workers' fatigue and exhaustion. This time-point defines the 'maximum acceptable working time' (MAWT) (Wu \& Wang, 2002) which is the maximum time that a worker can work without the appearance of exhaustion effects.

Using this methodology Wu and Wang have managed to estimate the maximum acceptable working time corresponding to each possible workload intensity. They have statistically calculated a mathematical relationship between workload intensity and maximum acceptable working time. We will use these results and embody them in the feedback relationship that we proposed earlier, for the following reasons. Firstly, because the above researchers assume a stable intensity of labour throughout the working day. As mentioned before, the intra-day variation of intensity is not the crucial aspect (although the feedback relationship can embody this case as well), because in many work processes intensity has to be stable and because the intra-day variation cannot explain the overall product decline when working time is rising. Secondly, they calculate a relationship that connects working time and intensity adversely. Every increase in intensity leads to a decrease in maximum acceptable working time. Thirdly and most importantly, an increase in working time is accompanied by a decrease in the intensity for the whole working day and not only for the time period of the increase, which is the crucial aspect of our approach.

The relationship provided by Wu and Wang (2002) (which is the latest development on this topic according to Drain et al 2016) between working time and workload intensity (for manual work), with the use of RHR index, is the following (the deterministic part, without the error term):

$$
\begin{equation*}
\text { MAWT }=26.12 \cdot \mathrm{e}^{-4.81 \cdot \mathrm{RHR}}, \quad\left(\text { with } \mathrm{R}^{2}=0.87\right) \tag{3}
\end{equation*}
$$

Where MAWT is the maximum acceptable working time and RHR is a proxy for workload intensity, as explained earlier. Since they have kept intensity stable during the 'working' period, their notion of intensity can be approximated with the notion of average intensity that we have used in equation (1). Under this assumption, solving equation (3) for intensity gives:

$$
\begin{equation*}
\varepsilon(t)=(3.262701-\ln (t)) / 4.81 \tag{4}
\end{equation*}
$$

Equation 4 is represented in figure 4


Figure 4. The estimated relationship among average intensity (proxy) and working time
Source: Own estimations using eq. (4) based on Wu \& Wang (2002)

If instead of the index RHR we use the index $\mathrm{RVO}_{2}$ as the proxy for intensity, the estimated equation is altered to:

$$
\begin{equation*}
\varepsilon(t)=(3.632309-\ln (t)) / 6.36 \tag{5}
\end{equation*}
$$

Equations 4 and 5 describe a relationship between maximum working time and maximum feasible workload intensity, for manual labour. We emphasize the word maximum, meaning that every shorter working time is of course feasible for a given intensity and every lower intensity is feasible for a given working time. These equations describe the contradiction between working time and intensity, or the rate of exchange of intensity in terms of working time.

At this point it must be noted that these indices ( RHR and $\mathrm{RVO}_{2}$ ) are used by all of the above mentioned authors to measure workload intensity for human organism and to approximate real labour intensity in terms of work done. They don't directly measure the work done, i.e. the velocity of labour movements, or the power of 'work' measured in Watts, but they are closely correlated to them. Unfortunately, Wu and Wang (2002) (like all the other authors mentioned above) don't provide evidence for the real work intensity in a form of a mathematical function, but only provide estimations for four separate points of labour intensity, in Table 2 (p.285) of their article. According to these points, with the help of our calculations and of table 3 (p.285) that correlates the three indices to real work intensity, it is evident that the RHR index is more closely related to the real work intensity and this is the reason for the use of only this index hereafter.

It must be noted that the above experiments did not take into account the negative psychological effects of working long hours, since they were not designed for long time periods. If that effects were to be considered it is expected that the reduction in productivity - intensity of labour would have been even greater for the long working hours, resulting in even smaller working time that maximizes output.

Finally, the above experiments were conducted with the help of young men and women (from 20 to 30 years of age, see Wu \& Wang, 2002), without carrying any load. The literature (see Drain et al, 2016 for a review) accepts that age, as well as carrying a load diminishes human capability to work. So we expect that the encounter of those additional factors would sharpen the relationship between working time and intensity, but
since we cannot calculate it specifically we don't take it into account. After all, we are obliged to adapt our model to the existing ergonometric experiments hoping that some experiments will be conducted according to our model in the future. Until then, the estimations made by us based on the existing ergo-metric experiments can only serve as an approximation of the true real relationship between working time and intensity.

### 4.1. Consequences to output-product

Given that the time - (maximum) intensity relationship for manual work is quantified, with the assumptions made above, the effect of working time on output-product of labour can now be estimated. First, the relationship between working time, intensity and output has to be determined. The term 'labour intensity' or 'work intensity' is sometimes connected with the notions of workers' performance, or efficiency. Green (2001) considers the performance of each worker as a function of his skills and effort, with the later to be of our interest. He argues that intensity is connected with the velocity of the workers' movements and with the pores of the working day (following Marx). In addition, one of the four ways ${ }^{2}$ that has been used to measure the intensity of labour (the Percentage Utilization of Labour methodology proposed by Bennet \& Smith-Gavine, 1987) measures workers' movements' velocity. Under this definition of labour intensity and under the assumption that all the other factors (mainly the technology of production and the organization of labour process) that affect productivity of the worker remain stable, the notions of intensity and productivity can be used equivalently. It must be noticed that we abstracted from the other consequences of working time (such as the capital operation time or employment), since our focus is on the main effect. We argue that the main effect of working time is on output, since this effect is on a higher level of abstraction, the sphere of production (for a methodological foundation see Fine and Harris, 1979 and for a more detailed analysis see loannides \& Mavroudeas, 2011).

Under the assumption of the average intensity used previously, total output is equal to the product of working time times the (average) intensity of labour (after adjusting measurement units):

$$
\begin{equation*}
Q(t)=\varepsilon(t) \cdot t \tag{6}
\end{equation*}
$$

Where $Q(t)$ is output, $\varepsilon(t)$ is the (daily average) intensity, $t$ is daily working time.
Labour intensity is a function of working time too, according to the feedback relationship of equation (4). So,

$$
\begin{equation*}
Q(t)=\varepsilon(t) \cdot t=\frac{3,262701-\ln (t)}{4,81} \cdot t \tag{7}
\end{equation*}
$$

Equation 7 connects output to working time for manual labour, on the basis of the working time - labour intensity exchange that was described by equation 4. The reservations made for the one equation hold for the other as well. Still, it is the best approximation available. Equation 7 describes output as a function of working time only if

[^2]for each working time the intensity worked was the maximum possible, i.e. if the managerial control of labour was very effective. It is an extreme case which sets the frontier of output. It can be easily calculated from equation 7 that output is maximized when working time equals 9,6 hours $\left(t^{*}=9,6\right.$ hours $)$


Figure 5. The relationship between working time and maximum output for manual labour
Source: Own estimations

If working time exceeds $\mathrm{t}^{\star}$, the output will not rise but will decline, as figure 5 shows. If working time (being greater then $\mathrm{t}^{*}$ ) is reduced, output will not be reduced but will be increased. Separating the effect of each working hour per day, from the first to the $15^{\text {th }}$ the results (presented on Table 1) are even more impressive.

Table 1. Effect of working time on output

| Working Time <br> (Hours per day) | Output <br> (adjusted units) | Output as percentage of maximum <br> output (\%) |
| :---: | :---: | :---: |
|  | RHR proxy | RHR proxy |$|$| 34,0 |
| :---: |
| 1 |

Source: Own estimations

It must be noted that the output units are consistent with the assumptions about intensity and output made above, so they do not identify with any specific measures of output and for this reason they are applicable to any kind of production.

We observe that the maximum value that output can take is 1,997 units and it is produced for 9,6 hours of work per day. When working time equals ten hours a day, output is reduced compared to the maximum and this reduction shall be greater the greater the working time becomes. In column 3 one can observe output as a percentage of the maximum possible output of 1,997 units. It is important to remember that the intensity of labour is assumed stable along a single working day. So the output observed in column 2 is the output that would have been produced if the working day was as long as it is indicated in column 1. For example, if the working day was 6 hours long the output of this working day would be 1,835 units and compared to the maximum possible output ( 1,997 units) it constitutes the $91,9 \%$ of it. As observed, the rise in output is small for each added working hour when working hours are long. This is not because this extra hour is less productive, but because all working hours become less productive due to the addition of this extra hour. It is impressive that from a seven to an eighth hour workday only $2,6 \%$ of the maximum output is added. More than this, if working day was 6 hours long, the output loss would have been only $6,7 \%$ of the output that is produced in a typical eight-hour day.

So, our basic results are that output elasticity towards working time is very small for working times above 5 or 6 hours a day and becomes negative for working time that exceeds 9.6 hours a day. These results are very similar to those of Cette et al (2011) and Dolton et al. (2016) presented above, who find that the output-product is diminishing when working time exceeds approximately eight and eight and a half hours a day respectively, despite the completely different methodologies used by them compared to us. We support that with the help of two different methodologies can verify an important outcome: There is no much economic sense on working long hours, especially for the workers, but for the enterprises as well. The implications of the above are striking for working time determination as well as for the consequences of working time in economy.

We also have to add that our estimations come from an experiment on young adults, without carrying load. Taking these factors into account is very probable that the working time that maximizes output is even smaller. All the long-term effects on workers health could have the same effect as well. And all this, without taking into account the psychological effect of long hours on workers, since this experiment was not designed to account for that. The above are reasons to make us believe that the working time that maximizes output could be even shorter than the 9,6 hours estimated above, but this has to be further examined.

It must be re-emphasized that the above relationship between (manual) working time and labour intensity and the subsequent one between working time and output describe only the frontier of these magnitudes; the maximum sustainable combinations that are viable. A smaller intensity for the same working time or less working time for the same intensity is of course sustainable, so the output would be smaller in this case. If we are dealing with jobs where the intensity of manual labour can be measured and controlled easily, then we expect workers to work close to their physical limits; that is close to the frontier described above. In these cases, the increase of working time beyond 8 hours is ineffective in terms of output. The opposite might have been more efficient, that is, a decrease in working time hours could lead to an output increase, not mentioning the satisfaction of the workers and thus the possible extra productivity gains. In these cases, equation 7 seems to verify the beliefs of both Marx, Chapman, and Robbins that there might be an output increase and a profit for capitalists if working time decreased towards 10 hours or less, given the fact that during their times 12 or 16 hour workdays prevailed. It
was the factory environment with the extensive use of machines that provided the capability of a constantly increasing intensity of manual labour and thus making the decrease of working time profitable.

Some conditions are different in our days where an increasing portion of jobs are on the tertiary sector. These kinds of jobs either pose objective limitations on intensity that are well below maximum, or make it very difficult for supervisors to monitor and control workers' intensity. Additionally, many of these jobs are not manual but mental. In the feedback equation 5 , assuming lower intensities than humanly maximum is leading to longer working hours that maximize output. So it is not surprising that the decreasing trend of working time seems to be inverted. For a detailed investigation of this subject it is necessary to embody the feedback relationship to the basic structure of economic models. Obviously this is a topic for further investigation.

An equally important topic for further investigation is the determination of a similar working time - maximum intensity relationship for mental labour as well. Finally, the aspect of the continuous nature of work needs to be investigated further.

## 5. Expansions of the model

The above analysis is an attempt to quantify the relationship between working time and intensity. It is valid for manual work only (since there are no such experiments for mental work) and it describes only the maximum values that the two dimensions of labour can take. This analysis (as well as those of Chapman, Barzel and Ehrenberg) is 'materially based', because it is founded upon the physical limitations of human body and mind. It was clearly in the spirit of the above authors, that with the help of the machines and the advanced systems of labour control, manual workers are forced to their limits. Nevertheless, this is not the whole story in our days. Upon the material base of the relationship between the maximum values of working time and intensity come the subjective, ethical, social and political factors that affect intensity determination, given the working time. As it was mentioned above, many economists have argued that workers' effort discretion plays a decisive role in labour intensity determination (e.g. Akerlof, 1982; Beyer \& Sorensen, 1991; Leibenstein, 1979). From a collective point of view, some authors (e.g. Green 2001) argue that there exists a social process of intensity determination. Workers collectively and atypically determine a 'fair working day', adjusting the length and the intensity of their workday and avoiding being extra squeezed. Class struggle and unions in a more organized way can also influence labour intensity. We have to remember that it was Marx who referred to a socially determined usual intensity of labour, although in many chapters of 'Capital' he argued that it is the 'human boundaries' that crucially affect the relationship between intensity and working time (For a further discussion on this topic, see loannides \& Mavroudeas, 2011).

We argue that all these 'willingness based' approaches are compatible with our approach. It is our belief that a materially based approach can provide a solid ground for understanding the individual or social procedures of intensity determination. Since further discussion on this issue cannot be done at this point, we can only make some comments. Most of the theories that try to explain intensity determination on the basis of workers' discretion acknowledge that there are limits to this discretion. They also acknowledge that there is a trade-off between working time and intensity; i.e., when working time increases, other things equal, the 'worker determined' intensity of labour is expected to decrease. This trade-off is defined by workers discretion. We maintain that the main reason for the
existence of this trade-off is the fact that working time and work intensity are the two dimensions of the consumption of labour power and so, when one of them increases workers try to decrease the other. So, it is the material basis of the contradiction between working time and intensity that mainly drives workers' mind when they choose the level of their effort. This material basis is described by a relationship like the one extracted earlier for manual labour. Even if we take account of workers' discretion, the 'real-world' relationship between time and intensity is expected to be analogous to the feedback relationship presented above.

## 6. Conclusions

The determination of maximum intensity given the working time (or else the determination of the relationship between working time and intensity) is a crucial matter for economic analysis but a very hard one to achieve. The latter is perhaps the reason that it was neglected. We make a step towards this direction by exploring the material base of the above relationship, for manual labour only. We have used ergometric data and modelled them in a specific way in order to extract a quantitative relationship, which still remains an approximation of the truth relationship, but a good one, according to the available data. The estimated relationship is based on the secular boundaries of human body. It does not tell the full story of intensity determination, but it constitutes a material basis upon which intensity determination is conducted.

The results of using this relationship in economic theory can be very important. We have only sketched some of them, mainly around the issue of working time determination. It is clear that the continuous and unlimited raising of working time might have finally negative effects on total product-output of the society. Any increase in manual working time beyond approximately 9 hours (and most probably less for the reasons mentioned before) might have negative results on output, at least in cases where the intensity of labour can be controlled efficiently. Even an increase from 6 to 8 hours seems almost useless in terms of output. So it seems that Marx, but also Chapman and Robbins were correct. The reduction of working time that occurred during their era, accompanied with the subsequent intensity increase, was leading to an increase in output. The historic trend of working time reduction might have served the same purpose as well.

Further analysis is needed in order to describe how workers' discretion, class struggle and other social factors interact with the feedback relationship in the final intensity determination. But it is very probable that the results don't alter radically when these factors are accounted for.

The step towards a quantitative estimation of the relationship between work time and intensity for all different kinds of work processes (and not only for manual work) can provide interesting insights to economic analysis. Especially for theoretical paradigms that attribute the principal role to labour, the implications can be even more important, since the interaction of working time with intensity alters the role of working time in the valueformation process. Many other topics, both theoretical and practical can be examined under the light of the relationship between working time and intensity.

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[^1]:    ${ }^{1}$ Some of the first studies that established this fact are Schor (1991); Leete \& Schor (1994); Bluestone \& Rose, 2000. Since then working time trends have not altered radically.

[^2]:    ${ }^{2}$ For a review see Green (2001).

